Rossmoyne Senior High School

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures | | | | | | | | | |

In words _____

_____

Your name _____

Circle your Teacher's Name:

Mr Adams | Mr Buckland | Mr Fletcher
Ms Leonard | Mr Pisano | Mr Tanday
Mr Younger

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
8	7	
9	7	
10	8	
11	7	
12	13	
13	8	
14	6	
15	10	
16	7	
17	8	
18	9	
19	8	
S2 Total	98	
S2 Wt (×0.6633)	65%	

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

See next page

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Supplementary page **-** This page has intentionally left blank.

Question number: _____

Question 8 (7 marks)

Mark has started an emu farm down in the Southwest of Western Australia. Each year he does a stocktake of the number of birds he has on the property. The population of birds he had for the first three years is shown in the table below. The number of birds is rounded to the nearest whole number.

Year	Start Population	End population
1	85	72
2	72	61
3	61	52

(a) Show that the recurrence relation for the above data is
$P_{n+1} = 0.85\ P_n \qquad P_0 = 85$. (2 marks)

(b) Describe the change in the population of birds for each year. (2 marks)

(c) Determine the number of birds he will have at the end of five years. (1 mark)

(d) After five years, Mark decides to contact his friend who has a similar farm and is willing to give Mark ten birds per year. By finding the steady–state solution, determine the effect this will have on Mark's bird population in the long term. (2 marks)

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Question 9 **(7 marks)**

(a) The monthly units of electricity u consumed by each apartment in a building was strongly associated with the average monthly maximum temperature, T °C. The least-squares line for the variables was $\hat{u} = 88.5 + 2.7T$.

(i) Predict the units of electricity consumed by an apartment in a month when the average monthly maximum temperature was 33°C. (1 mark)

(ii) In a month when the average monthly maximum temperature was 25°C, an apartment consumed 153.4 units of electricity. Calculate the residual for this data point. (2 marks)

(b) In a government study, the correlation coefficient for the association between age and superannuation balance for employed adults was found to be 0.755. What percentage of the variation in superannuation balance for employed adults is unexplained by their variation in age? (2 marks)

(c) After measuring the age and hearing acuity of a group of pensioners, a researcher observed a negative linear association between the variables and found that 61% of the variation in hearing acuity can be explained by the variation in age. Determine the correlation coefficient for the association. (2 marks)
(acuity is the ability to hear sounds)

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Question 10 **(8 marks)**

A random sample of adults who were not working and not seeking work were recently asked for the main reason that they were not looking for work. The responses, categorised by the sex of the adult and their main reason, are summarised in the table below.

Reason	Male	Female
Education	143	116
Family considerations	33	117
Other	84	37

(a) How many adults gave a response? (1 mark)

(b) What percentage of the females gave education as their main reason? (2 marks)

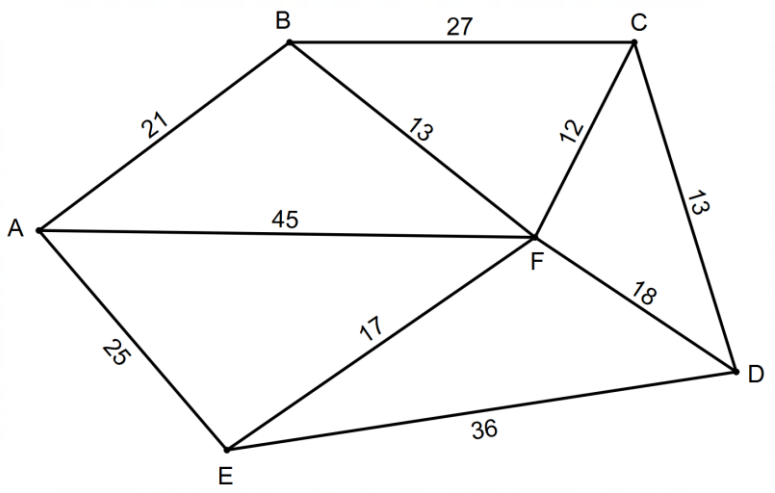
(c) Construct a table showing column percentages for the above data, rounding entries to the nearest whole number. (3 marks)

(d) Discuss whether the data from the survey suggests the presence of an association between the variables sex and reason. (2 marks)

See next page

Question 11 **(7 marks)**

The graph below represents a cycle pathway around a city. Each weight represents kilometres.



(a) Determine the shortest time and path to travel from A to D. (2 marks)

(b) Suggest a possible Hamiltonian Cycle. (2 marks)

Any paths that are longer than 25 kilometres are going to be closed due to the high maintenance costs.

(c) (i) State which roads will be closed. (1 mark)

(ii) When the paths are closed, all the remaining paths need be cleaned, by a mechanical sweeper, in one continuous cycle without going over the same path. State a trail starting from E. (2 marks)

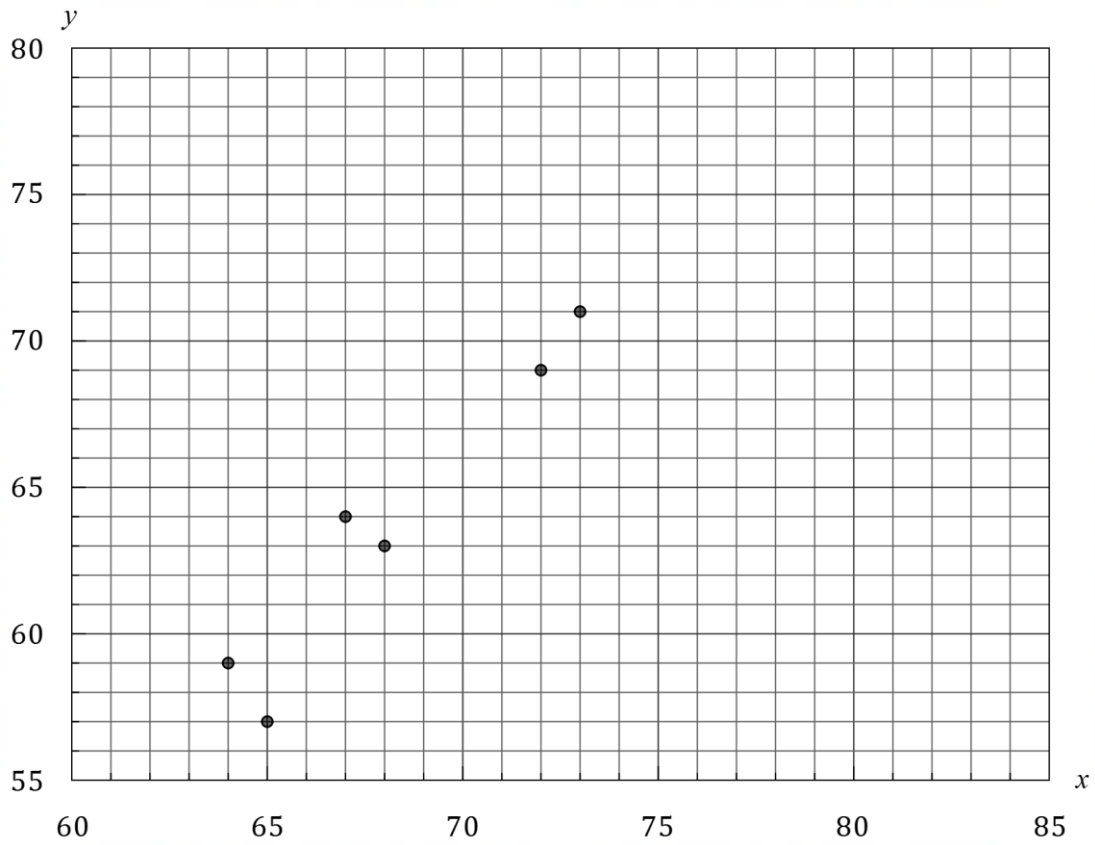
See next page

Question 12 **(13 marks)**

The table below shows the life expectancy, in years, of females and males in nine countries in Oceania.

Country	Female (x)	Male (y)
Federated States of Micronesia	72	69
Kiribati	64	59
Marshall Islands	73	71
Nauru	65	57
New Caledonia	80	74
Palau	78	68
Papua New Guinea	68	63
Solomon Islands	74	67
Tuvalu	67	64

(a) On the scatterplot below, plot the three missing data points from the table. (2 marks)



(b) Determine the coefficient of determination between the variables and interpret its value in the context of the question. (2 marks)

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(c) State the correlation coefficient between the variables and use its value to comment on the strength of the linear association between female and male life expectancy for these countries. (2 marks)

(d) Determine the equation of the least-squares line to model the relationship between the variables and draw this line on the scatterplot. (3 marks)

(e) The life expectancy of a female from Fiji is 70. Predict, to the nearest year, the life expectancy of a male from the same country and comment on any factors that affect the validity of your prediction. (2 marks)

(f) The life expectancy of a female Australian is 86. Predict, to the nearest year, the life expectancy of a male Australian and comment on any factors that affect the validity of your prediction. (2 marks)

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Question 13 **(8 marks)**

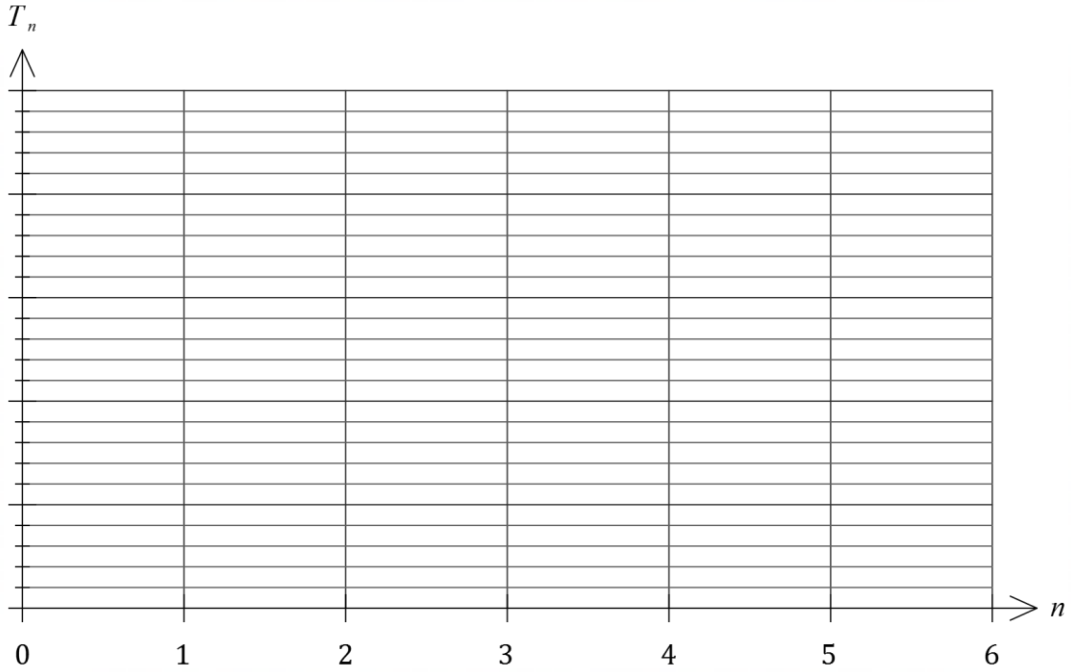
The cooling system for a mobile cool room has just been turned on. The temperature T_n °C inside the cool room, n hours later, is modelled by the linear recurrence relation

$$T_n = 0.74T_n + 0.65, \qquad T_0 = 24.$$

(a) Complete the table of temperatures below. (2 marks)

n	0	1	2	3	4	5	6
T_n (°C)	24.0	18.4					

(b) Add a scale to the vertical axis below and then plot the temperature inside the cool room every hour. (3 marks)



(c) After how many hours does the model predict that the temperature inside the cool room will first reach within $0.1°$ of its steady state? Justify your answer. (3 marks)

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Question 14 **(6 marks)**

The mass of a small puppy was measured as 375 g when it was one week old. A week later its mass had increased by 45 g.

(a) Assuming that the weekly mass of the puppy can be modelled by an arithmetic sequence, predict the mass of the puppy when it is 9 weeks old. (2 marks)

(b) Assuming that the weekly mass of the puppy can be modelled by a geometric sequence, predict the mass of the puppy when it is 9 weeks old. (3 marks)

(c) Comment on the usefulness of these models as the puppy gets older. (1 mark)

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Question 15 **(10 marks)**

An industrial chemist varied the amount of accelerant (a grams) used when making an epoxy resin and recorded the time taken (t seconds) for the resin to set. Some of the results are shown below.

a	4.5	5.5	6.5	7.0	8.0	9.0	10.0	11.5	13.0	14.0
t	24.1	19.2	19.3	21.8	15.7	19.2	14.8	17.7	15.0	12.3

The chemist suspected that a linear association might exist between the variables and calculated the correlation coefficient $r_{at} = -0.81$.

(a) After seeing this value of the correlation coefficient, the chemist said to their assistant "it looks like there is a strong and negative linear association between the variables". Explain why the chemist made this statement. (2 marks)

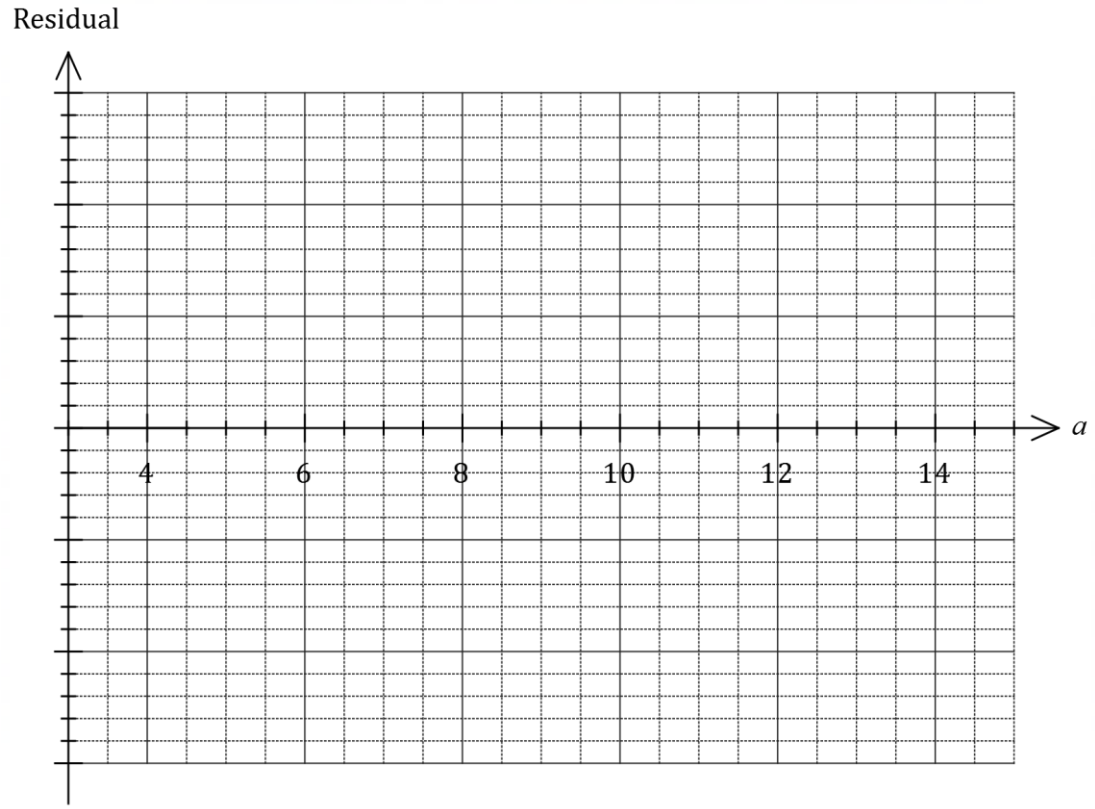
The chemist also noted that the least-squares line for the data was $\hat{t} = 25.78 - 0.896a$ and used it to calculate nine residuals for the linear model as shown below, rounded to one decimal place.

a	4.5	5.5	6.5	7.0	8.0	9.0	10.0	11.5	13.0	14.0
Residual	2.3	-1.7	-1.7		-2.9	1.5	-2.0	2.2	0.9	-0.9

(b) Show how the residual of -2.9 was calculated and determine the residual associated with 7.0 grams of accelerant. (3 marks)

See next page

(c) Construct a residual plot for the data on the axes below. (3 marks)



(d) Does the residual plot support the chemist's suspicions that a linear model fits the data? Explain your answer. (2 marks)

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Question 16 **(7 marks)**

A business bought a mainframe computer valued at $\$95\,000$. The value of the computer depreciated by 35% each year.

(a) By how much did the value of the computer depreciate during the first year and what was its value one year after it was bought? (2 marks)

(b) Deduce a recursive rule for V_n, the value of the computer after n years. (2 marks)

(c) Calculate the value of the computer after 4 years. (1 mark)

(d) During which year does the value of the computer **first depreciate** by less than $\$1000$? Justify your answer. (2 marks)

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Question 17 **(8 marks)**

The annual number of mobile phone subscriptions and new cars sold in Western Australia, as collated by a researcher, are shown in the table below.

Year	2011	2012	2013	2014	2015	2016	2017
Subscriptions (s, in millions)	2.93	3.01	3.06	3.14	3.19	3.25	3.31
New cars (c, in thousands)	109	111	112	114	116	117	119

The researcher wanted to identify whether new car sales in Western Australia could be predicted from mobile phone subscriptions.

(a) Quantify the strength of the linear association between the variables s and c. (1 mark)

(b) Determine the equation of the least-squares line that can be used to predict c from s. (2 marks)

(c) Use the least-squares line to predict the number of new car sales in another Australian state that had 3.09 million mobile phone subscriptions, and comment, with reasons, on the validity of your prediction. (3 marks)

(d) Describe a possible non-causal explanation for the observed association between mobile phone subscriptions and new cars sold. (2 marks)

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Question 18 **(9 marks)**

The adjacency matrix for the connected planar graph P is $\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$.

(a) Determine, with justification, the number of faces that P has. (3 marks)

(b) Use elements from the adjacency matrix to explain why P is a simple graph. (3 marks)

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(c) Ore's theorem states that a simple graph with n vertices is Hamiltonian if, for every pair of distinct vertices V_a and V_b which are not adjacent, the sum of the degrees of V_a and V_b is greater than or equal to n. Use Ore's theorem to show that P is Hamiltonian. (3 marks)

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Question 19 **(8 marks)**

John is a rower who rows four times during a month. At the end of each month he increases the distance rowed by a ratio and then adds a constant amount to establish his goal for the following month. In the first month, he rowed a distance of 5 kilometres each time he rowed. In the second month, the distance increased to 5600 metres for each row and for the third month he was rowing 6260 metres for each row.

(a) Write the first order recurrence relation for the above data. (3 marks)

(b) Determine how many kilometres he would row during the fifth month. (2 marks)

(c) Once John reaches ten kilometres he will stop increasing the distance.
How many months of training does John complete before first reaching this level? (1 mark)

(d) Determine the total distance John has rowed after five months of training. (2 marks)

End of questions

Supplementary page

Question number: _____